Synthesis Tree Sorting: Insert

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Theory

Multisets: Properties of Union

Proposition (Union-Associativity)

In[72]:= $\left(\forall_{A}\left(\forall_{B}\left(\forall_{C}\left(((A \uplus B) \uplus C) == (A \uplus B \uplus C)\right)\right)\right)\right)$ (union − associativiy)

Proposition (Union-Commutativity)

In[73]:= $\left(\forall_{A}\left(\forall_{B}\left((A \uplus B) == (B \uplus A)\right)\right)\right)$ (union − commutativity)

Proposition (Union-Unit)

In[74]:= $\left(\forall_{A}\left((A \uplus \emptyset) == A\right)\right)$ (union − unit)

Trees: Ordering

Proposition (Smaller-than-Empty-Tree)

In[75]:= $\left(\forall_{x} x \leq \epsilon\right)$ (smaller − than − empty − tree)

Proposition (Empty-Tree-Smaller-than)

In[76]:= $\left(\underset{x}{\forall}\ \epsilon \leq x\right)$ (empty – tree – smaller – than)

Proposition (Tree-Less-Elem)

In[77]:= $\left(\underset{L}{\forall}\left(\underset{a}{\forall}\left(\underset{R}{\forall}\left(\underset{b}{\forall}\left(\left(\langle L, a, R\rangle \leq b\right) \Leftrightarrow \left(L \leq b \wedge a \leq b \wedge R \leq b\right)\right)\right)\right)\right)\right)$ (tree – less – elem)

Proposition (Elem-Less-Tree)

In[78]:= $\left(\underset{L}{\forall}\left(\underset{a}{\forall}\left(\underset{R}{\forall}\left(\underset{b}{\forall}\left(\left(b \leq \langle L, a, R\rangle\right) \Leftrightarrow \left(b \leq L \wedge b \leq a \wedge b \leq R\right)\right)\right)\right)\right)\right)$ (elem – less – tree)

Trees: Multisets

Definition (Full-Tree-Multiset-Direct)

In[79]:= $\left(\underset{L}{\forall}\left(\underset{a}{\forall}\left(\underset{R}{\forall}\left(\mathcal{M}[\langle L, a, R\rangle] == \left(\mathcal{M}[L] \uplus \{\{a\}\} \uplus \mathcal{M}[R]\right)\right)\right)\right)\right)$ (full – tree – multiset – direct)

Proposition (Full-Tree-Multiset-Reverse)

In[80]:= $\left(\underset{L}{\forall}\left(\underset{a}{\forall}\left(\underset{R}{\forall}\left(\mathcal{M}[\langle L, a, R\rangle] == \left(\mathcal{M}[R] \uplus \{\{a\}\} \uplus \mathcal{M}[L]\right)\right)\right)\right)\right)$ (full – tree – multiset – reverse)

Proposition (Unit-Tree-Multiset)

In[81]:= $\left(\underset{a}{\forall}\left(\mathcal{M}[\langle \epsilon, a, \epsilon\rangle] == \{\{a\}\}\right)\right)$ (unit – tree – multiset)

DEFINITION (EMPTY-TREE-MULTISET)

In[82]:= $\left(\mathcal{M}[\epsilon] == \emptyset\right)$ (empty – tree – multiset)

Trees: Sorting

DEFINITION (SORTED-EMPTY-TREE)

In[83]:= IsSorted[ϵ] (sorted – empty – tree)

DEFINITION (SORTED-TREE)

In[84]:= $\left(\underset{L}{\forall}\left(\underset{a}{\forall}\left(\underset{R}{\forall}\left(\text{IsSorted}[\langle L, a, R\rangle] \Leftrightarrow \left(\text{IsSorted}[L] \wedge \text{IsSorted}[R] \wedge L \leq a \wedge a \leq R\right)\right)\right)\right)\right)$ (sorted – tree)

Conjectures

CONJECTURE (INSERT-SORTED)

In[85]:= $\left(\underset{X}{\forall}\left(\text{IsSorted}[X] \Rightarrow \left(\underset{a}{\forall}\left(\left(\left(\mathcal{M}[\text{Insert}[a, X]] == \{\{a\}\} \uplus \mathcal{M}[X]\right) \wedge \text{IsSorted}[\text{Insert}[a, X]]\right)\right)\right)\right)\right)$ (insert – sorted)